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1. (10 points) Let $A_{n\times n}, x_{n\times 1}$, $\delta^{x,b}_{n\times 1}$, $b_{n\times 1}$, and $\delta^{b}_{n\times 1}$ be real valued matrices. Define what is meant by, $\kappa(A)$, the condition number of A. Let $W_{n\times n}, x_{n\times 1}, b_{n\times 1}$ be matrices. Assume that it is known that $\kappa(A)$ is large and κ is not. Decide whether the fact that A is ill-conditioned prevents solving $Ax = b$ upto a reasonable error.

2. (10 points) Manually construct the divided-difference table for interpolating degree- 3 Newton polynomial for the following data set:

x	-1	0	1	2
h(x)	2	5	8	-1

Further construct the Newton's polynomial.

3. (10 points) Let

$$B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 0 \\ 4 & 2 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & -1 & 2 \end{bmatrix}$$

(a) Compute $A = BC$

(b) Determine a basis for the column-space of A and justify that this is a basis.

(c) Does the system $Ax = b$ have a solution for the vector $b = [0 \quad 0 \quad 1 \quad 1]^T$? If so, does it have one solution or infinitely many. *Answer without using Gaussian Elimination on A and justify your answers*

4. (15 points) Use the composite Simpson rule with $h = 1$ to estimate

$$\int_{-1}^{3} x^2 dx.$$

Verify that this gives the exact value of the integral with no error and explain why there is no error.

5. (15 points) Suppose

$$S(x) = \begin{cases} a + b(x-1) + c(x-1)^2 + d(x-1)^3 & x \in [0,1] \\ (x-1)^3 + ex^2 - 1 & x \in [1,2] \end{cases}$$

and

$$T(x) = \begin{cases} x^2 + x^3 & x \in [0,1] \\ a + bx + cx^2 + dx^3 & x \in [1,2] \end{cases}$$

(i) Determine the parameters a, b, c, d and e so that S is a cubic spline interpolation satisfying the natural end conditions.
(ii) Determine the parameters a, b, c and d so that T is a cubic spline interpolation satisfying $T'''(2) = 12$ conditions.

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6. (30 points)

(a) Please write down (1) - (5) that are left blank in the following program:

```
function [t,y] = odeRK4(diffeq,tn,h,y0)
% odeRK4  Fourth order Runge-Kutta method for a single, first order ODE
%
% Synopsis:  [t,y] = odeRK4(fun,tn,h,y0)
%
% Input:     diffeq = (string) name of the m-file that evaluates the right
%                     hand side of the ODE written in standard form
%            tn  = stopping value of the independent variable
%            h   = stepsize for advancing the independent variable
%            y0  = initial condition for the dependent variable
%
% Output:    t = vector of independent variable values:  t(j) = (j-1)*h
%            y = vector of numerical solution values at the t(j)

t = (0:h:tn)';                  %  Column vector of elements with spacing h
n = length(t);                  %  Number of elements in the t vector
y = y0*ones(n,1);               %  Preallocate y for speed
h2 = h/2;  h3 = h/3;  h6 = h/6;  %  Avoid repeated evaluation of constants

%  Begin RK4 integration; j=1 for initial condition
for j=2:n
   k1 =  -----------(1)----------- ;
   k2 = ------------(2)------------;
   k3 = ------------(3)------------;
   k4 = ------------(4)------------;
   y(j)= -----------(5)------------;
end
```

QUESTION CONTINUES TO PAGE 3

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(b) Please write down (7) - (15) that are left blank in the following program:

```
function xm = demoBisect(xleft,xright,n)
% demoBisect  Use bisection to find the root of x - x^(1/3) - 2
%
% Synopsis: x = demoBisect(xleft,xright)
%           x = demoBisect(xleft,xright,n)
%
%  Input:   xleft,xright = left and right brackets of the root
%           n = (optional) number of iterations;  default:  n = 15
%
%  Output:  x = estimate of the root

if nargin<3, n=15;  end    %  Default number of iterations
a = -----(6)---;   b = ------(7)------;   %  Copy original bracket to local variables
fa = ------(8)--------;      %  Initial values of f(a) and f(b)
fb =--------(9)--------;
fprintf('  k       a            xmid          b             f(xmid)\n');

for k=1:n
  xm = -------(10)-----;      %  computing the midpoint
  fm = -------(11)-----;      %  f(x) at midpoint
  fprintf('%3d  %12.8f  %12.8f  %12.8f  %12.3e\n',k,a,xm,b,fm);
  if sign(fm)==sign(fa)
    a = --------(12)-----;
    fa = -------(13)-----;
  else
    b = ----(14)-----;
    fb = ---(15)-----;
  end
end
```

QUESTION CONTINUES TO PAGE 4

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(c) Please write down (16) - (21) that are left blank in the following program:

```
function [c,R2] = linefit(x,y)
% linefit    Least-squares fit of data to y = c(1)*x + c(2)
%
% Synopsis:   c      = linefit(x,y)
%            [c,R2] = linefit(x,y)
%
% Input:   x,y = vectors of independent and dependent variables
%
% Output:  c  = vector of slope, c(1), and intercept, c(2) of least sq. line fit
%          R2 = (optional) coefficient of determination; 0 <= R2 <= 1
%               R2 close to 1 indicates a strong relationship between y and x
if length(y)~= length(x),  error('x and y are not compatible');  end

x = ----(16)-----;  y = ----(17)----;    %  Make sure that x and y are column vectors
A = ---------(18)--------;  %  m-by-n matrix of overdetermined system
c = ---------(19)--------;      %  Solve normal equations
if nargout>1
  r = -----(20)-----------;
  R2 =-----(21)-----------;
end
```